Abstract
Ongoing assessment and effective cost reduction for the transport of goods is one of the basic tasks of a management in companies. The paper presents the approach to transport costs reduction by establishing of a distribution warehouse in an appropriate location. Suitable location of a distribution warehouse is realized by applying three geometrical methods – by using axial distance, quadratic distance and by using Cooper's iterative method, based on the selected coordinate system. There are compared results and calculated objective function for the place of allocation and evaluated reduced transport costs at the end of the paper.

Keywords: transport cost, distribution warehouse, allocation calculations.

1. INTRODUCTION
Ongoing assessment and effective cost reduction for the transport of goods is one of the basic tasks of a management in companies. The costs of products transport play an important role also in determining the competitiveness of a company, because they represent a significant proportion of costs to the total production (Rosová, 2007). These costs are largely determined by the distances between producers and consumers. The transport costs are relatively considerable and are also affected by road infrastructure and geographical situation. These two factors have an impact on the number of kilometers traveled with the load. One of the possibilities of effective and efficient reduction of these costs is to establish a distribution warehouse. Producers may choose whether to use of existing public warehouses in logistics centers or to build its own warehouse. Each option has pros and cons (Šaderová, 2014). The key decision is to choose a suitable location - allocation of a warehouse.
The paper is organized as follows: Section 2 gives an overview of posts related to this topic.

In Section 3 is described a methodology of a warehouse allocation by using geometric methods. In Section 4 is application of methodology. Section 5 and 6 presents the results and discussion. Section 7 concludes the work.

2. LITERATURE REVIEW

Allocation of a warehouse should be executed based on the assessment of several factors - economic or non-economic (Straka, 2013). The most often there are many factors, which should be taken into account to a site selection: distance of customers, the amount of transported goods, type of transport technology, capacity of means of transport, infrastructure of transport routes etc. (Fabianová & Janeková, 2015), (Andrejová et al., 2015).

There are two main approaches used for the allocation of companies and thus their warehouses in new markets - multi-criteria evaluation and a geometric approach in many references. Multi-criteria evaluation of problem solving is covered by several authors who describe methods for determining values of the criteria and the methods for calculating overall usefulness of particular options (Ocelíková et al., 2005). The paper (Stopka et al., 2014) aims at identification of appropriate multi-criteria analysis methods for allocation tasks of logistics objects in a certain area. The paper includes an overview of utilized methods for determining criteria weightings and methods of selection of the most suitable variant. The methods differ mainly according to how criteria values are determined (Straka et al, 2015), too.

Geometric methods are based on a different principle (Straka, 2013). The effort of these methods is to calculate the coordinates of a warehouse for its optimal location within a given area (region). Geometric methods are based on maps of the region in which we want to allocate the warehouse and on the coordinates x and y. The literature provides several types of geometrical methods and their applications (Malindžák & Straka, 2003) (Šaderová et al., 2015).

3. METHODOLOGICAL BASIS FOR ALLOCATING A WAREHOUSE USING GEOMETRIC METHODS

First of all it is important to rely on the location of existing company customers when allocating a warehouse. Therefore, the first step is the allocation of customers into the map of the region at which it is considerate a warehouse location with the coordinate axis x, y. Every place that enters the evaluation or has some links to our search is given its coordinates [x, y] in the coordinate system of the region. The
second step is to determine the objective function which costs should be minimized. The objective function is described by the transport costs TC - Transport Cost (Straka, 2012):

$$\min z = TC = \sum_{i=1}^{n} M_i C_i d_i$$  \hspace{1cm} (1)$$

where

$M_i$ – quantity transported (material, products, energy),

$C_i$ – shipping (transport) price for the $i$-th material,

$d_i$ – distance from the customer and distribution center,

$n$ – number of distribution points entering to the calculation.

The objective function follows needs, it may also cover other criteria such as unit price, quantity of goods, number of devices and number of vehicles or alternatively choice from the quantity or the unit price only. The distance $d_i$ has a great importance in these methods and must be chosen depending on the region where the location of the warehouse is looking for. Depending on the region character, the distance $d_i$ can be expressed as:

distance between axes  
$$d_i = |x - x_i| + |y - y_i|$$  \hspace{1cm} (2)$$

quadratic distance  
$$d_i = (x - x_i)^2 + (y - y_i)^2$$  \hspace{1cm} (3)$$

direct distance  
$$d_i = \sqrt{(x - x_i)^2 + (y - y_i)^2}$$  \hspace{1cm} (4)$$

direct corrected distance  
$$d_i = k \sqrt{(x - x_i)^2 + (y - y_i)^2}$$  \hspace{1cm} (5)$$

where:

$[x,y]$ – coordinates of the searched distribution center,

$[x_i,y_i]$ – coordinates of distribution points in the distribution region,

$i$ – index of distribution points entering to the calculation, it takes the values 1,2,3 .. $n$,

$n$ – number of distribution points entering to the calculation,

$k$ – correction value, usually 1.2.

Three selected geometrical methods are used for calculation of an allocation of distribution warehouse (Straka, 2012):
A. Allocation using axial distance. Application of this method was done in following steps:

1. distribution points that are part of the solution were arranged in ascending order according to the coordinates $x_i$, (in Table 2).

2. for each location were calculated cumulative values $M C_i$, (Table 2).

3. the $x$-coordinate to be searched for distribution warehouse allocation will be equal to that coordinate $x_i$, in which the cumulated value of $M C_i$ exceeds as the first the value of $\frac{1}{2} \sum_{i=1}^{n} M C_i$.

4. distribution points that are part of the solution were sorted according to ascending order of coordinates $y_i$, (Table 3).

5. for each location were calculated cumulative values $M C_i$, (Table 3).

6. the $y$-coordinate to be searched for distribution warehouse allocation will be equal to that coordinate $y_i$, in which the cumulated value of $M C_i$ exceeds as the first the value of $\frac{1}{2} \sum_{i=1}^{n} M C_i$.

B. Allocation using quadratic distance. Values of $x$ and $y$ coordinates for distribution warehouse using a quadratic distance can be calculated as follows:

$$x = \frac{\sum_{i=1}^{n} M C_i x_i}{\sum_{i=1}^{n} M C_i} \quad (6)$$

$$y = \frac{\sum_{i=1}^{n} M C_i y_i}{\sum_{i=1}^{n} M C_i} \quad (7)$$

The city which is closest to that coordinates becomes a suitable location to allocate the distribution warehouse.

B. Allocation using direct distance, Cooper’s Iterative Method. The Cooper’s iterative method is used to calculate the optimal coordinates for distribution warehouse allocation so that the city which is the closest to that coordinates becomes a good place for allocating a distribution warehouse. Coordinates calculations were carried out as described in the literature (Malindžák & Straka, 2003).
4. APPLICATION OF THE SELECTED GEOMETRIC METHODS FOR ALLOCATING OF A WAREHOUSE

Warehouse allocation was performed according to the following steps:

1. Determination of input parameters to the solution.
2. The selection of the coordinate system.
3. Location of customers in the selected coordinate system.
4. The application of the geometric methods (GM).
5. Evaluation of the results of GM.
6. Calculation of the objective function.

The company located in eastern Slovakia delivers its products to customers within the whole Slovakia and the most important customers are shown in the Figure 1. The company has decided to create a study how to reduce transportation costs by establishing of a warehouse in the region of Central Slovakia. Customers from central and western Slovakia would be supplied from the established warehouse. Customers from Poprad, Košice and Trebišov would be supplied from the company headquarter.

Figure 1 shows customers and areas that are eligible for allocation of distribution warehouse and points coordinates in a coordinate system WGS 84 (World Geodetic System) in nonagezimal terms.
Three geometrical methods are used to calculate an allocation of distribution warehouse for given conditions:

1. Allocation by using axial distance.
2. Allocation by using quadratic distance.
3. Allocation by using direct distance, Cooper’s Iterative Method.

Table 1 lists the input data for the solution such as: the amount of received goods, transport price per unit of goods and the points coordinates in a coordinate system WGS 84 (World Geodetic System) in nonagezimal terms. In Table 1 the value of the x coordinate is $\lambda$ - geodetic longitude referred to a Greenwich and y-coordinate is $\phi$ - geodetic latitude.

<table>
<thead>
<tr>
<th>Allocation alternatives for distribution warehouses</th>
<th>M [pcs]</th>
<th>C [€]</th>
<th>Customer coordinates x ($\lambda$)</th>
<th>y ($\phi$)</th>
<th>M.Ci</th>
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<td>1.84</td>
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Allocation by using axial distance. In Table 2 there are data for allocation using axial distance to the coordinates $x_i$ and in Table 3 there are data for allocation using axial distance to the coordinates $y_i$.

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Search coordinates $x, y$ for allocation of distribution warehouse is that one where the cumulated value $M_iC_i$ has exceeded as first the value \[ \sum_{i=1}^{n} M_iC_i = 20268.14/2 = 10134.07 \].

The coordinate according to the Table 2 is $x = \lambda = 18.0752^\circ$E.

The coordinate according to the Table 3 is $y = \varphi = 48.7327^\circ$N.

Allocation by using quadratic distance. Values of $x$ and $y$ coordinates for distribution warehouse using a quadratic distance can be calculated to the relations (6) and (7). The coordinates $[18.5704^\circ$E $48.6032^\circ$N] were calculated for the given conditions.

Allocation by Cooper’s Iterative Method. Allocation of the distribution warehouse was calculated with an accuracy of $\varepsilon = 0.1$ according to the $x$-axis. The initial coordinates had a values $[x(0) y(0)] = [0,0]$. It is required to calculate a coordinates $[x(1) y(1)]$ to achieve optimal coordinates $[x(opt) y(opt)]$. After determining the coordinates it is required to check whether they meet the required accuracy $\varepsilon$. If the condition is not satisfied, the calculation proceeds to the next iteration.

The optimal coordinates $[x(opt) y(opt)]$ are $[18.4782^\circ$E $48.5843^\circ$N] after 3rd iteration.

**5. RESULTS**

We have got a 3 sets of coordinates by the three methods that we have used.

1. By method of axial distance we got $[18.0752^\circ$E $48.7327^\circ$N].
2. By method of quadratic distance we got $[18.5704^\circ$E $48.6032^\circ$N].
3. By Cooper’s iterative methods we got $[18.4782^\circ$E $48.5843^\circ$N].

In Figure 2 there are highlighted the locations whose coordinates are calculated by above-mentioned methods.

Based on the marking of coordinates, they create a triangle among locations: Banská Bystrica, Nitra and Nové Mesto nad Váhom.
There are graphically displayed results of each method in the Figure 3, it means suitable localities for warehouse location - the nearest location to the calculated coordinates.

The coordinates obtained by the first method are closest to the area of Nové Mesto nad Váhom. The coordinates obtained by the other method are closest to the area of Banská Bystrica. And coordinates obtained by the third method are approximately in the same distance from all three sites.

6. DISCUSSIONS

Based on these facts, it can be concluded that a warehouse may be established either in the area of Nové Mesto nad Váhom or in the area of Banská Bystrica. The selection of the final locality will be determined by the value of objective function TC for the locality: $TC_{\text{Banská Bystrica}} = 21974.68$ or $TC_{\text{Nové Mesto nad Váhom}} = 127837.2$. 
Based on these results we can conclude that a suitable location for the distribution warehouse allocation seems to be a city of Banská Bystrica or its surroundings (periphery).

Figure 5 shows the percentage ratio of travelled kilometres from the headquarters to the warehouse set up in Banská Bystrica, in the case of cumulating of the original number of journeys from 5 to 30%. For the given conditions, the total transport costs from the headquarters to a customer at cumulated run about 30% could be decreased from € 1.37 / m² to 1.03 € / m².

7. CONCLUSIONS

Allocation of distribution warehouse is not a final point for company. The allocation of distribution warehouse is followed by other important decisions concerning the building renting or leasing the warehouse in the chosen region, its size-capacity, decisions regarding the selection of warehouse equipment (shelves, handling equipment), personnel, warehouse management and the way of physical distribution of goods to customers (Straka, 2010). All these decisions are influenced by several factors that need to be taken into account. Before its execution, it is important to perform simulation (Fedorko et al., 2014) a financial analysis of decisions, (Rosová, 2010), their impact on the company, to develop forecasts of market developments (Kačmáry & Malindžák, 2010) as well as verify the decisions by creating a virtual network using proper software applications (Saniuk et al., 2014).

The company can reduce costs by using other possibilities (Čulková et al., 2015). The following possibilities are about decreasing of fuel costs, which belongs to the most costly items. One of the possibilities is to find out cheaper supplier or to purchase in greater volumes for better price. The second option is about the fuel cards given to drivers, these may give the manufacturer information.
about fuel with the best price. An interesting option for the company is to use software application with geographical distance of customers and the possible directions of goods supply. The transport costs are also influenced by the age of a car fleet. The modern cars save not only costs of fuel, tolls, service and maintenance of trucks but also our environment. Then, one of the last options may be the transport outsourcing.

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REFERENCES


